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MOND predictions of 'halo' phenomenology in disc galaxies

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ABSTRACT

We examine two corollaries of MOND pertaining to properties of the equivalent dark matter halo. MOND predicts for pure exponential discs a tight relation involving the halo and disc scalelengths and the mean acceleration in the disc, which we find to test favourably against the Verheijen sample of Ursa Major galaxies. A correlation between halo and disc length-scales is also apparent when the 'maximum disc' contribution is assumed, but we demonstrate that this follows from the more general MOND prediction. The second MOND prediction involves the existence of a maximum halo acceleration, which also tests favourably against the Ursa Major sample for different assumptions on the disc contribution.

Key words: galaxies: kinematics and dynamics – dark matter.

1 INTRODUCTION

The quintessential test of Modified Newtonian Dynamics (MOND) involves full analysis of rotation curves of disc galaxies. MOND prescribes a simple formula whereby the mass discrepancy in galaxies – as reflected in their measured rotation curve – can be determined from the visible (baryonic) mass distribution alone. For those who opt to view MOND as only a very useful summary of dark matter properties – a description that 'saves the phenomena' but is not underpinned by new physics – this means that the distribution of baryons determines that of the dark matter (DM). This sweeping prediction has been tested successfully in many galaxies as reviewed e.g. by Sanders & McGaugh (2002), but in it are buried a number of corollaries that deserve separate discussion and testing. These take the form of various predicted properties of 'dark haloes', correlations between visible matter and 'dark matter' attributes, etc. In MOND all these result from one equation with one universal constant, as Newtonian dynamics explain the different regularities in planetary motions. Without the umbrella of MOND these bylaws would appear as a set of many independent 'Kepler laws' of galactic dynamics; see e.g. Milgrom (2002) for details.

It is useful to consider such predictions in their own right because (1) they deal with more limited properties of the mass discrepancy and hence are easier to take in; (2) detailing them brings home the richness and variety of the predictions of MOND; (3) they constitute intermediate challenges for dark matter theories, which have, so far, anything but dealt with the tight relation between baryonic and DM distributions encapsulated in MOND.

An example of such a phenomenological bylaw brought to light by MOND is the predicted correlation between rotational velocity and *total* baryonic mass of a galaxy: the baryonic Tully–Fisher (TF) relation. The innovation introduced by MOND is the inclusion of the total visible mass comprising that in gas as well as in stars, as stressed by Milgrom & Braun (1988). It differs from the standard TF relation, which involves only starlight, and hence only stellar mass. This has been tested by McGaugh et al. (2000) (and see also McGaugh 2004). Another corollary concerns the predicted shape of the DM distribution in disc galaxies (Milgrom 2001). Yet another is the MOND prediction of the absence of a mass discrepancy in the body of elliptical galaxies with high central surface densities (Milgrom & Sanders 2003).

Here we discuss and test two more such phenomenological laws predicted by MOND, which, for pedagogical reasons, we express as properties of an equivalent fictitious dark halo. The first regularity, examined in Section 2 for the first time, concerns a correlation between the disc and 'halo' scalelengths. The second prediction, discussed in Section 3, was noted by Brada & Milgrom (1999) but has not been tested before. It states that there is an absolute maximum acceleration that a 'dark halo' can produce.

2 CORRELATION BETWEEN HALO AND DISC SCALELENGTHS

Donato & Salucci (2004) have recently found that in a sample of disc galaxies, selected from several different sources on the basis of the accuracy of the measured rotation curve, the core radius of the best-fitting pseudo-isothermal (PI) halo, R_c, correlates well with the scalelength of the stellar disc, R_d. The correlation that they find is of the form $R_c \approx 2R_d$. The claimed tightness of the correlation is somewhat surprising as halo core radii are notoriously difficult to pin down. Because of the well known disc–halo degeneracy, when analysing the inner parts of rotation curves, the required halo core radii are usually rather poorly determined without further assumptions on the contribution of the disc (Verheijen 1997; Verheijen et al. 2004). There are several possible ways of breaking this degeneracy:

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for example, one may assume a 'reasonable' mass-to-light ratio for the stellar disc, or that the disc makes its maximum possible contribution to the rotation curve in the inner regions – the so called 'maximum disc hypothesis' (van Albada & Sancisi 1986). Donato & Salucci took the fitted halo parameters directly from the cited sources and used in some cases the maximum-disc fits, while in other cases the fits were made with an assumed mass-to-light ratio (M/L) for the disc, and for several objects the unconstrained minimum χ^2 fits were used. So, the role of these various assumptions on the claimed correlation could not be readily assessed.

We have thus checked the correlation on another independent, more homogeneous sample, the Ursa Major spirals observed by Verheijen (1997). Although the H I rotation curves are not all of the highest quality, there are several advantages to using this sample: all galaxies are roughly at the same distance (no relative distance uncertainty), and there exists near-infrared (K' band) surface photometry of all objects. The near-infrared is less susceptible to effects of dust obscuration and recent star formation, thus eliminating an important source of uncertainly in the determination of both R_c and R_d. In any event, insistence on inclusion only of galaxies with very accurate rotation curves is somewhat misplaced, as by far the most prominent source of uncertainty in the present context is the above-noted disc–halo degeneracy. We have, none the less, excluded from the sample those galaxies designated by Verheijen as 'kinematically disturbed'.

As noted by Verheijen, when fitting pseudo-isothermal haloes to the rotation curves, χ^2 minimization using all three parameters indicates a very broad and noisy minimum. We therefore made separate halo fits to the entire sample applying four different assumptions about the disc contribution: (i) maximum-disc; (ii) theoretical $M/L_{K'}$ values obtained from the observed colours (Bell et al. 2003); (iii) an assumed constant $M/L_{K'}$(=0.9), which is approximately what the theoretical results give; and (iv) $M/L_{K'}$ values determined by MOND one-parameter fits (Sanders & Verheijen 1998). The number of objects (18–22) included differs in the various cases because it was not possible to achieve fits, using the PI halo, to all objects with, for example, disc masses from fixed M/L.

We plot in Fig. 1 the fitted halo core radius obtained under these four assumptions versus the exponential-disc scalelength. It is evident that the only procedure giving an apparent correlation is the maximum-disc assumption (correlation coefficient ≈0.7).

The work of Donato & Salucci has prompted us to check what MOND says about such a correlation. Because in MOND the 'halo' mass distribution is determined by that of the visible matter, the scalelength of the 'halo', in particular, depends on that of the baryonic galaxy. However, there are other important factors that enter, which vary from galaxy to galaxy, so we do not expect a sharp correlation (as, for example, is expected in the baryonic TF relation). In the first place, the 'halo' scalelength depends also on the mass distribution in the galaxy: on the exact shape of the stellar mass distribution, on the relative contribution of gas and its distribution, and on the presence and relative contribution of a bulge. Even if we consider a sample of homologous baryonic galaxies, such as pure exponential discs, there remains an important disc parameter on which halo properties depend: the mean acceleration in the galaxy in units of the MOND constant a_0.

For the purpose of demonstration we shall, indeed, concentrate hereafter on pure exponential discs. For these we use, as in Milgrom (1983), the parameter $\xi \equiv v_\infty^2/R_d a_0 = (MG/a_0 {R_d}^2)^{1/2}$ as the measures of the mean acceleration, where here R_d is the exponential scalelength, v_∞ is the asymptotic rotational speed, and M is the total mass (ξ is also a measure of the mean surface density in units of a_0/G). We then define the 'halo' as the PI mass density distribution, $\rho(r) = \rho_0 {R_c}^2/\left({R_c}^2 + r^2\right)$, that, combined with the disc, produces a Newtonian rotation curve that best matches the MOND rotation curve of the disc alone. The ratio $Q \equiv R_c/R_d$ can then be shown to depend on ξ alone: $Q = Q(\xi)$.

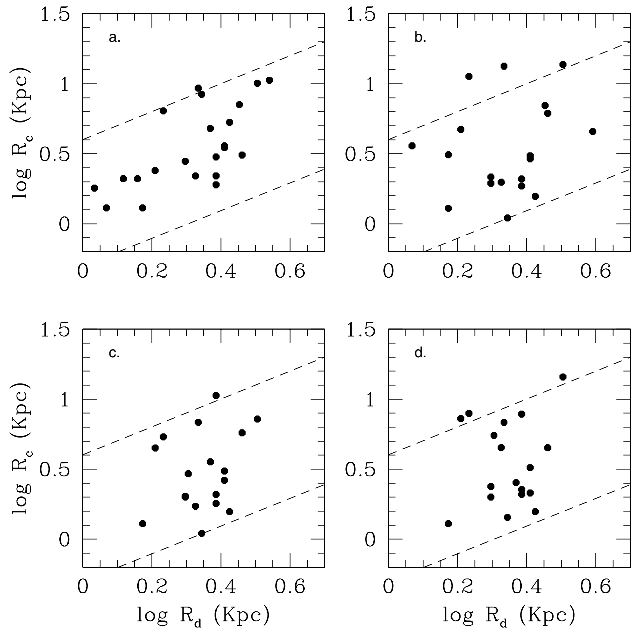

Figure 1. Core radius of the best-fitting isothermal sphere plotted against the exponential scalelength of the disc for Ursa Major spirals. The different panels correspond to different assumptions on the mass-to-light ratio of the stellar disc: (a) maximum disc; (b) M/L from population synthesis models (Bell et al. 2003) and the measured galaxy $B - R$ colours (Verheijen 1997); (c) constant M/L'_K (=0.9); (d) best value from MOND fits. The parallel dashed lines indicate $R_c/R_d = 0.5$, 4.0.

In the limit of $\xi \ll 1$, i.e. deep in the MOND regime, the MOND scaling laws dictate that R_c becomes proportional to the R_d. For exponential discs completed with PI haloes we find numerically in this limit $Q \approx 0.5$. As ξ increases beyond $\sim$1 the inner parts of the disc become increasingly Newtonian, the relative contribution of the halo there decreases, and thus its core radius increases relative to R_d. In the limit of high ξ, $Q(\xi) = q\xi$. This is true (with the same q) for any galactic mass distribution (as long as it is bound) since in this limit the 'halo' enters only at large radii where the galaxy may be taken as a point mass; and for a point mass $R_c \propto r_t$, where $r_t \equiv (MG/a_0)^{1/2} = \xi R_d$ is the transition radius. The constant q is found numerically to be $q \approx 0.9$. A plot of $Q(\xi)$ for exponential discs is shown in Fig. 2 (assuming a MOND interpolating function of the form $\mu = x/\sqrt{1 + x^2}$).

Galaxy discs have ξ values that do not exceed $\sim$4. This is related to the generalized Freeman law whereby the distribution of mean (or central) surface brightness of galactic discs is cut off from above at a value that corresponds, when converted to a mean mass surface density, to $\sim a_0 G^{-1}$. At the other end, there are low surface brightness galaxies for which $\xi < 1$. So we expect Q to range from 0.5 to $\sim$4 – as indeed is seen to be the case in Fig. 1 for all but one of the assumptions on M/L. The distribution of Q for the maximum-disc assumption is narrower (see below). So, rather than a tight correlation between R_c and R_d, we expect on the basis of MOND (for homologous discs) a 'fundamental surface' in the space (R_c, R_d, ξ) given by $R_c = R_d Q(\xi)$.

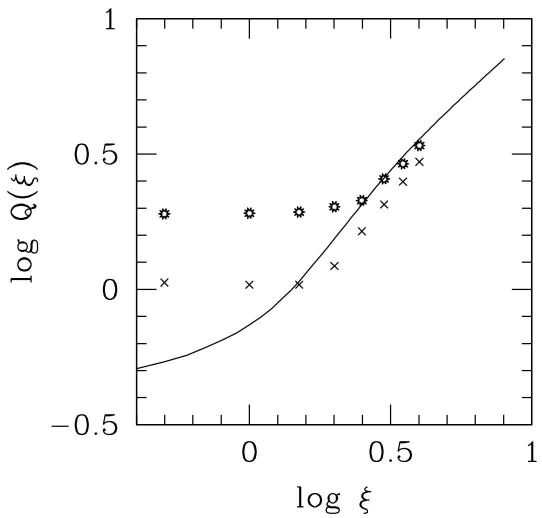

Figure 2. The solid curve shows the MOND deduced ratio $Q \equiv R_c/R_d$ as a function of the mean acceleration parameter ξ for pure exponential discs. The crosses and open points show, respectively, the result for minimum χ^2 and maximum-disc fits to MOND rotation curves of exponential discs assuming Newtonian dynamics and a PI dark halo.

Again, we test this prediction on the Ursa Major sample. Most of the UMa galaxies are well represented by exponential discs (Tully & Verheijen 1997). An exception to this is NGC 3992 – a barred spiral that does have a bulge, and the total light distribution of which is far from exponential. It was thus excluded from the sample.

We use the halo properties determined with the four different choices of the stellar M/L values considered above. The results are shown in Fig. 3. *Note that the solid line is not a fit to this correlation, but the MOND prediction for the equivalent PI haloes*. The respective correlation coefficients are 0.48, 0.82, 0.77 and 0.88. One might argue that the use of MOND disc masses (Fig. 3d) is not an independent test of MOND predictions, but it is evident that estimating the disc contribution from population synthesis M/Ls or even a constant M/L of 0.9 (in the K' band) yields the expected correlation. These three assumptions on M/L are based on some prior theoretical concept. We know from previous analysis that MOND M/L values are in good agreement with theoretical values, and that both give a narrow distribution for the K band. So it is not surprising, in light of this previous knowledge, that all three give results consistent with each other. The maximum-disc assumption, while frequently applied, is rather arbitrary, and is the one that gives a poor agreement with the MOND prediction. This is expected in MOND: only galaxies with high ξ are expected to be correctly fitted by a maximum disc.

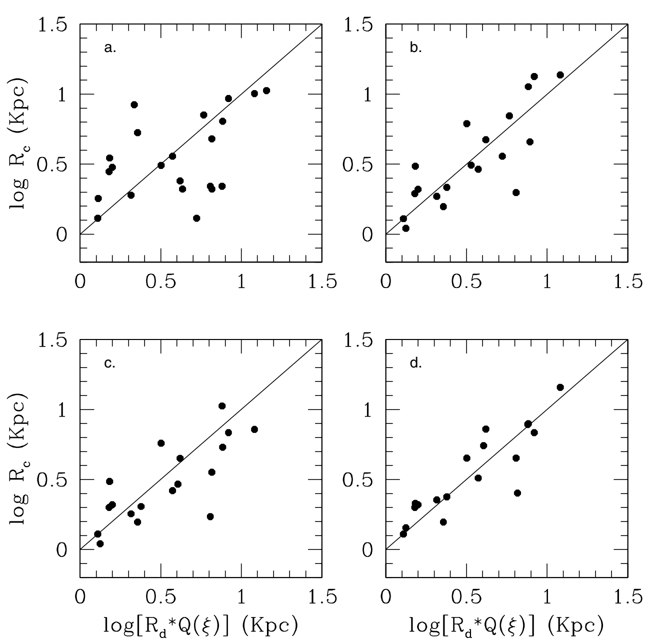

Figure 3. The halo core radius, R_c, deduced by making different assumptions on the stellar M/L values (as in the corresponding panels in Fig. 1), plotted against $R_d Q(\xi)$. The solid line is the MOND prediction.

The question remains: Why do Donato & Salucci obtain a good correlation between R_d and R_c without the scatter expected for a wide distribution of ξ, as we do for the Ursa Major sample when we apply the maximum-disc assumption? We believe that the reason rests in the following. The MOND 'halo', for $\xi < 2$, is not typically well described by an isothermal sphere. Thus the best-fitting halo obtained by simultaneously fitting for the disc M/L and the halo parameters (the common practice) does not give the same halo as obtained from the best MOND fit. To simulate what is done in Newtonian fits we should not take the MOND 'halo' and fit it with an PI, but should take the full MOND rotation curve and perform the conventional Newtonian fit on it. This we did for exponential discs with a wide range of ξ values. We also applied a Newtonian maximum-disc fit to this set. We calculated the Q value obtained from these as a function of ξ. The results are shown by the points in Fig. 2, where the crosses show minimum χ^2 fits and the open points show maximum-disc fits. It is evident that, in both cases, Q saturates at 1 and 2 respectively for $\xi < 3$. Thus the range of Q predicted by MOND for maximum-disc fits, for example, is considerably narrower than when using the correct disc contribution. The same is true to a lesser extent for the best (three-parameter) fits. This explains, in terms of MOND, the correlation found by Donato & Salucci: the apparent relation $R_c \approx 2R_d$ is an artefact of the fitting procedure which typically assumes maximum disc and therefore artificially narrows the distribution of Q.

3 MAXIMUM HALO ACCELERATION

Brada & Milgrom (1999) have pointed out that the MOND 'halo' acceleration in the plane of the galactic disc cannot exceed a maximum value of the order of a_0. This results from the relation

$$g_h(g) = g - g_N = g - g\mu(g/a_0)$$

between the true (MOND) acceleration, g, and the halo acceleration, g_h (excess MOND over Newtonian), where g_N is the Newtonian acceleration, and μ is the interpolating function of MOND. This expression is exact in MOND as modified inertia (Milgrom 1994), but is only approximate in modified gravity MOND such as the formulation described by Bekenstein & Milgrom (1984). In any event, barring an anomalous behaviour of $\mu(x)$, g_h cannot exceed a value of the order of a_0, call it $a_{max} = \eta a_0$, where η is constant of the theory (including the exact choice of the function μ). For the forms of μ used standardly $\eta \approx 0.3$–0.4.

Such a maximum halo acceleration, if verified observationally, would constitute a challenge for dark matter theories. McGaugh (2004) has recently demonstrated, in fact, that the above limit is inconsistent with the Navarro, Frenk & White (1996, NFW) halo parameters deduced from cold dark matter N-body simulations, which give too high halo accelerations for high surface density galaxies.

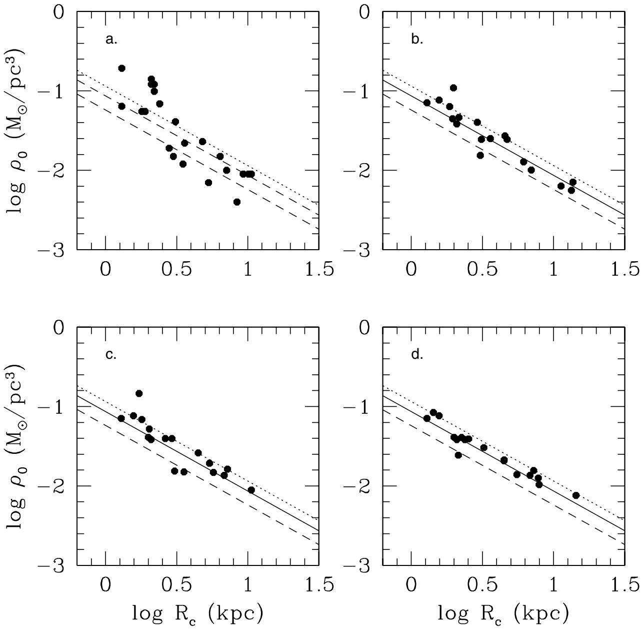

Figure 4. The fitted halo central density (ρ_0) plotted against the halo core radius, R_c (log–log plot), where the disc contribution is constrained via the four assumptions applied previously (Figs 1 and 3). The parallel lines correspond to maximum halo accelerations, the dashed line being $0.2a_0$, the solid line $0.3a_0$ and the dotted line $0.4a_0$.

We test the maximum halo acceleration on best-fitting PI haloes. For such haloes the acceleration goes to zero at small and large radii, and reaches a maximum at $\approx 1.5R_c$, where its value is

$$a_{max} \approx 2.9G\rho_0 R_c.$$

Again we consider the PI fits to UMa spirals with the four assumptions on the stellar contribution as in Figs 1 and 3. A plot of the resulting central halo density, ρ_0 versus the halo core radius, R_c, is shown in Fig. 4 along with lines of constant maximum halo acceleration given by the expression above. The $\pm 1\sigma$ errors on the fitted core radius and density are typically 30 and 50 per cent respectively, but these errors are correlated: the error ellipses are extended roughly parallel to the lines of constant $\rho_c R_c^{1.5}$. None the less, it is evident that the equivalent PI haloes do seem to exhibit a characteristic maximum acceleration ($g_h < 0.4a_0$) as predicted by MOND. The higher accelerations evident for several objects in the upper left panel of Fig. 4, the maximum disc case, are galaxies (e.g. NGC 4085) for which the maximum-disc assumption actually *underestimates* the disc contribution because of beam-smearing of the rotation curve in the inner regions; this leads to an over-estimate of the halo contribution.

5 CONCLUSIONS

The simple MOND formula makes many definite predictions of regularities in the properties of the equivalent, Newtonian PI haloes, two of which we have considered here. The first is that, when the galaxies are pure discs, and the surface density distribution is well described by a simple exponential, one expects a tight correlation involving the disc length-scale, the core radius and the mean acceleration parameter. The core radius is not expected to be well correlated to the disc radius for a sample that involves a spread in the mean acceleration, unless one makes an assumption, such as maximum disc, which artificially restricts the distribution of R_c/R_d. From Figs 1 and 3 we see that this prediction appears to be borne out for the UMa sample.

The second prediction is that there exists a maximum possible halo acceleration, which depends somewhat upon the form of the MOND interpolating function μ but is typically less than a_0. In Fig. 4 it is evident that fitted halo core radii and central densities are indeed such that the halo acceleration is always less than $\sim 0.4a_0$, as predicted.

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